Solution

Class 09 - Mathematics

MATHEMATICS

Section A

1. **(a)** 4

   Explanation: $3^x + 64 = 2^6 + (\sqrt{3})^8$
   But we know that,
   $2^6 = 64$
   So, $2^6 + 3^x = 2^6 + \left(\sqrt{3}\right)^{2\times 4}$
   $\Rightarrow 2^6 + 3^x = 2^6 + (3)^4$
   Now by equating both
   We get
   x = 4

2. **(d)** $\sqrt[56]{x}$

   Explanation: The seventh root of x divided by the eighth root of x , is
   Seventh root of x = $\sqrt[7]{x} = x^{\frac{1}{7}}$
   Eighth root of x = $\sqrt[8]{x} = x^{\frac{1}{8}}$
   The seventh root of x divided by the eighth root of x
   = $x^{\frac{1}{7}} \div x^{\frac{1}{8}}$
   =$x^{\frac{1}{7}-\frac{1}{8}}$
   = $x^{\frac{8-7}{56}}$
   = $x^{\frac{1}{56}}$
   = $\sqrt[56]{x}$

3. **(c)** 10

   Explanation: $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ + $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
   $\Rightarrow \frac{(\sqrt{3}+\sqrt{2})^2+(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}$
   $\Rightarrow \frac{(3+2+2\sqrt{6})+3+2-2\sqrt{6}}{3-2}$
   $\Rightarrow 10$

4. **(d)** $\frac{\sqrt{7}+2}{3}$

   Explanation: After rationalising:
   $\frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$
   =$\frac{\sqrt{7}+2}{(\sqrt{7})^2-(2)^2}$
   = $\frac{\sqrt{7}+2}{7-4}$
   = $\frac{\sqrt{7}+2}{3}$

5. **(b)** $\sqrt[3]{2}$

   Explanation: $\sqrt[3]{500}$
   = $\sqrt[3]{5 \times 2 \times 5 \times 2 \times 5}$
   = $\sqrt[3]{5 \times 5 \times 5 \times 2 \times 2}$
   =$5\sqrt[3]{2 \times 2}$
   =$5\sqrt[3]{4}$
   The simplest rationalising factor of $\sqrt[3]{500}$, is $\sqrt[3]{2}$

6. **(a)** positive and irrational

Explanation:
$$\left(5+\sqrt{8}\right)+\left(3-\sqrt{2}\right)\left(\sqrt{2}-6\right)$$
$$=\left(5+2\sqrt{2}\right)+(3\sqrt{2}-18-2+6\sqrt{2})$$
$$=\left(5+2\sqrt{2}\right)+(9\sqrt{2}-20)$$
$$=11\sqrt{2}-15$$

And we know that the value of 11√2 is greater than 15 so it's value will be positive,
And also sum or differences of rational and irrational is irrational

7. **(b)** 5

Explanation:
$$5^{\frac{1}{3}}\times5^{\frac{1}{3}}$$
$$=5^{\frac{2}{3}}\times5^{\frac{1}{3}}$$
$$=(5)^{\frac{2+1}{3}}\Longleftrightarrow5$$

8. **(d)** 25

Explanation:
$$(0.00032)^{\frac{-2}{5}}$$
$$=\left(\frac{32}{100000}\right)^{\frac{-2}{5}}$$
$$=\left(\frac{2}{10}\right)^{5\times\frac{-2}{5}}$$
$$=\left(\frac{1}{5}\right)^{-2}=25$$

9. **(a)** $\frac{4}{3}$

Explanation: $\frac{\sqrt{48}+\sqrt{32}}{\sqrt{27}+\sqrt{18}}$

= $\frac{\sqrt{4\times4\times3}+\sqrt{4\times4\times2}}{\sqrt{3\times3\times3}+\sqrt{3\times3\times2}}$

= $\frac{4\sqrt{3}+4\sqrt{2}}{3\sqrt{3}+3\sqrt{2}}$

= $\frac{4(\sqrt{3}+\sqrt{2})}{3(\sqrt{3}+\sqrt{2})}$

= $\frac{4}{3}$

10. **(b)** $3+2\sqrt{2}$

Explanation: After rationalising:

$\frac{1}{\sqrt{9}-\sqrt{8}}=\frac{1}{\sqrt{9}-\sqrt{8}}\times\frac{\sqrt{9}+\sqrt{8}}{\sqrt{9}+\sqrt{8}}$

= $\frac{\sqrt{9}+\sqrt{8}}{(\sqrt{9})^2-(\sqrt{8})^2}$

= $\frac{\sqrt{3\times3}+\sqrt{2\times2\times2}}{9-8}$

= $\frac{3+2\sqrt{2}}{1}$

= 3+ $2\sqrt{2}$

11. **(b)** 5

Explanation: $(x^3 - 2)(x^2 - 11)$

= $x^3(x^2 - 11) - 2(x^2 - 11)$

= $x^5 - 11x^3 - 2x^2 + 22$

Here the highest power is 5.
Therefore, the degree is 5.

12. **(d)** 497

Explanation: $(249)^2 - (248)^2$

= (249 + 248)(249 - 248) [Using identity $a^2 - b^2 = (a + b)(a - b)$]

= 497 × 1

= 497

13. **(a)** 108

Explanation: Given: $x+y+z=9$ and $xy+yz+zx=23$

$x^3+y^3+z^3-3xyz$ = $(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$

= $(x+y+z)\left[(x+y+z)^2-2xy-2yz-2zx-xy-yz-zx\right]$

= $(x+y+z)\left[(x+y+z)^2-3xy-3yz-3zx\right]$

= $(x+y+z)\left[(x+y+z)^2-3(xy+yz+zx)\right]$

= $(9)\left[(9)^2-3(23)\right]$

= $9\times[81-69]$

= 9 x 12

= 108

14. **(b)** n is an odd integer

Explanation: The linear polynomial (x - 1) is a factor of $x^n + 1$, only if

$f(-1) = (-1)^n + 1 = 0$

If n is odd integer, then f(-1) = -1 + 1 = 0

15. **(b)** $(x^4 - y^4)$

Explanation: $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)(x + y)(x^2 + y^2)$

= $\left[\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2\right]$ $(x + y)(x^2 + y^2)$

= $(x - y)(x + y)(x^2 + y^2)$

= $[(x)^2 - (y)^2](x^2 + y^2)$

= $(x^2 - y^2)(x^2 + y^2)$

= $(x^2)^2 - (y^2)^2$

= $x^4 - y^4$

16. **(b)** 21

Explanation: $x^4 + 2x^3 - 3x^2 + x - 1$

Using remainder theorem,

= $(2)^4 + 2(2)^3 - 3(2)^2 + 2 - 1$

= 16 + 16 - 12 + 2 - 1

= 34 - 13

= 21

17. **(b)** 3abc

Explanation: If $a+b+c=0$, then

$a^3+b^3+c^3-3abc=0$

$\Rightarrow a^3+b^3+c^3$ = $3abc$

18. **(c)** 5

Explanation: $x^3+\left(\frac{1}{x^3}\right)=110$

$x^3+\left(\frac{1}{x^3}\right)+3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)=110+3x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\Rightarrow\left(x+\frac{1}{x}\right)^3=110+3\left(x+\frac{1}{x}\right)$

$\Rightarrow\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)-110=0$

Let $x+\frac{1}{x}=a$

$\Rightarrow a^3 - 3a - 110 = 0$

$\Rightarrow a^3 - 5a^2 + 5a^2 - 25a + 22a - 110 = 0$

$\Rightarrow a^2(a - 5) + 5a(a - 5) + 22(a - 5) = 0$

$\Rightarrow (a - 5)(a^2 + 5a + 22) = 0$

$\Rightarrow$ a - 5 = 0 or $a^2 + 5a + 22 = 0$ neglected)

$\Rightarrow$ a=5
$\Rightarrow x + \frac{1}{x} = 5$

19. **(c)** 0

Explanation: p(x) = $x^3 - x^2 + x + 1$

= $\frac{p(-1)+p(1)}{2}$

= $\frac{(-1)^3-(-1)^2+(-1)+1+(1)^3-(1)^2+(1)+1}{2}$

= $\frac{-1-1-1+1+1-1+1+1}{2}$

= $\frac{0}{2}$

= 0

20. **(b)** $2y(3x^2 + y^2)$

Explanation: put a=x+y and b=x-y,then

$(x+y)^3 - (x-y)^3 = a^3 - b^3$
$= (a-b)(a^2 + b^2 + ab)$
$= (x+y-x+y)[(x+y)^2 + (x-y)^2 + (x-y)(x+y)]$
$= 2y[2(x^2+y^2) + (x^2-y^2)]$
$= 2y[3x^2 + y^2]$

Section B

21. Rationalizing factor of $\sqrt{5} - 2$ is $\sqrt{5} + 2$

22. We have,

$(2^3)^4 = (2^2)^X$

$\Rightarrow 2^{3\times4} = 2^{2\times x} \Rightarrow 2^{12} = 2^{2x}$

Comparing, we get

$2x = 12 \Rightarrow x = \frac{12}{2} = 6$

$\therefore$ x = 6

23. $\sqrt{\frac{9}{27}} = \frac{1}{\sqrt{3}}$, which is the quotient of a rational and an irrational number and therefore an irrational number.

24. The given number is rational number.

Justification:

Consider $(1+\sqrt{5}) - (4+\sqrt{5}) = 1 - 4 + \sqrt{5} - \sqrt{5}$

= 1 – 4

= 3, which is clearly a rational number.

Thus, $(1+\sqrt{5}) - (4+\sqrt{5})$ is rational.

25. We have,

$\left(\frac{64}{25}\right)^{\frac{-3}{2}} = \frac{1}{(\frac{64}{25})^{3/2}} = \frac{1}{\left\{\left(\frac{8}{5}\right)^2\right\}^{\frac{3}{2}}} = \frac{1}{\left(\frac{8}{5}\right)^{\frac{2\times3}{2}}}$

$= \frac{5\times5\times5}{8\times8\times8} = \frac{125}{512}$

26. $2^{x-7} \times 5^{x-4} = 1250$

$\Rightarrow 2^x \cdot 2^{-7} \cdot 5^x \cdot 5^{-4} = 2 \times 5 \times 5 \times 5 \times 5$ [since 1250 $=2 \times 5 \times 5 \times 5 \times 5$]

$\Rightarrow \frac{2^x \times 5^x}{2^7 \times 5^4} = 2 \times 5 \times 5 \times 5 \times 5$

$\Rightarrow (10)^x = 2^8 \times 5^8 = (10)^8$

Comparing both sides, we get

x = 8

27. The given expression may be rewritten as,

$5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$

[$\because 20 = 15 + 5$ and $15 \times 5 = 5\sqrt{5} \times 3\sqrt{5}$]

$= 5 \times (\sqrt{5}x + 3) + \sqrt{5}(\sqrt{5}x + 3)$

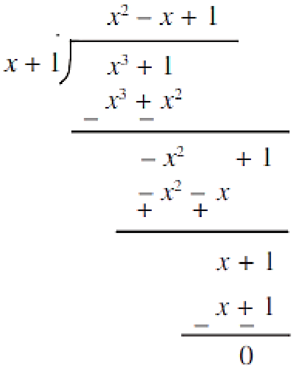
$= (\sqrt{5}x + 3)(5x + \sqrt{5})$

$\therefore 5\sqrt{5}x^2 + 20x + 3\sqrt{5} = (\sqrt{5}x + 3)(5x + \sqrt{5})$

28. $3\sqrt{t}+t\sqrt{2}$
We can observe that in the polynomial $3\sqrt{t}+t\sqrt{2}$ we have t as the only variable and the powers of t in each term are not a whole number.
Therefore, we conclude that $3\sqrt{t}+t\sqrt{2}$ is not a polynomial in one variable.
29. $f(x)=2x^2+7x+3$
$f(-2)=2(-2)^2+7(-2)+3$
= 8 -14 + 3 = 11 - 14 = -3
30. $\frac{x^2}{2}-\frac{2}{x^2}$
The given expression can be written as $\frac{x^2}{2}$ - $2x^{-2}$
It contains a term having negative integral power of x. Therefore, it is not a polynomial.
31. Let x = $0.\overline{54}$
⇒ x = 0.545454... ---(i)
Multiply eq. (i) by 100 we get,
⇒ 100 x = 54.5454 ... ---(ii)
subtracting eq. (i) from (ii) we get,
⇒ 100 x - x = 54.5454...- 0.5454...
⇒ 99 x = 54
⇒ x = 54/99
⇒ x = 6/11
Hence, $0.\overline{54}=\frac{6}{11}$
32. $\sqrt[5]{x^4\sqrt[4]{x^3\sqrt[3]{x^2\sqrt{x}}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^3\sqrt[3]{x^2\cdot x^{1/2}}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^3\sqrt[3]{x^{\left(2+\frac{1}{2}\right)}}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^3\sqrt[3]{x^{5/2}}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^3\cdot\left(x^{5/2}\right)^{1/3}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^3\cdot x^{5/6}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^{\left(3+\frac{5}{6}\right)}}}$
= $\sqrt[5]{x^4\sqrt[4]{x^{\left(\frac{23}{6}\right)}}}$
= $\sqrt[5]{x^4\cdot\left(x^{\frac{23}{6}}\right)^{\frac{1}{4}}}$
= $\sqrt[5]{x^4\cdot x^{\frac{23}{24}}}$
= $\sqrt[5]{x^{\left(4+\frac{23}{24}\right)}}$
= $\left(x^{\frac{119}{24}}\right)^{\frac{1}{5}}$
= $x^{\left(\frac{119}{24}\times\frac{1}{5}\right)}$
= $x^{\frac{119}{120}}$
33. We have
$\frac{6-4\sqrt{3}}{6+4\sqrt{3}}$
= $\frac{(6-4\sqrt{3})}{(6+4\sqrt{3})}\times\frac{(6-4\sqrt{3})}{(6-4\sqrt{3})}$
= $\frac{(6-4\sqrt{3})^2}{(6)^2-(4\sqrt{3})^2}$

$= \frac{6^2+(4\sqrt{3})^2-2\times6\times4\sqrt{3}}{36-48}$
$= \frac{36+48-48\sqrt{3}}{-12}$
$= \frac{84-48\sqrt{3}}{-12}$
$= \frac{48\sqrt{3}-84}{12}$
$= \left(4\sqrt{3}-7\right)$

34. $4^{x-1} \times (0.5)^{3-2x} = (\frac{1}{8})^x$
$\Rightarrow (2^2)^{x-1} \times (\frac{1}{2})^{3-2x} = (\frac{1}{2^3})^x$
$\Rightarrow 2^{2x-2} \times 2^{-3+2x} = 2^{-3x}$
$\Rightarrow 2^{2x-2-3+2x} = 2^{-3x}$
$\Rightarrow 2^{4x-5} = 2^{-3x}$
$\Rightarrow 4x - 5 = -3x$
$\Rightarrow 7x = 5$
$\Rightarrow x = \frac{5}{7}$

35. By long division,we have,

Therefore, remainder is 0.
Here p(x) = $x^3 + 1$, and the root of x + 1 = 0 is x = –1. We have,
$p(-1) = (-1)^3 + 1$
$= -1 + 1 = 0$,
which is equal to the remainder obtained by actual division.

36. $p(x) = x^3$
$\therefore p(0) = (0)^3 = 0$,
$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$

37. $g(x) = \frac{x}{3} - \frac{1}{4} = 0$ gives $x = \frac{3}{4}$
g(x) will be a factor of p(x) if $p\left(\frac{3}{4}\right) = 0$ (Factor theorem)
Now, $p\left(\frac{3}{4}\right) = 8\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{4}\right) + 3$
$= 8 \times \frac{27}{64} - 6 \times \frac{9}{16} - 3 + 3 = 0$
Therefore, g(x) is a factor of p(x).

38. $3\sqrt{48} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$
$=3\sqrt{16\times3} - \frac{5}{2}\sqrt{\frac{3}{3\times3}} + 4\sqrt{3}$
$== 3\times4\sqrt{3} - \frac{5}{2}\cdot\frac{1}{3}\sqrt{3} + 4\sqrt{3}$
$=12\sqrt{3} - \frac{5}{6}\sqrt{3} + 4\sqrt{3}$
$=\left(12 - \frac{5}{6} + 4\right)\sqrt{3} = \left(16 - \frac{5}{6}\right)\sqrt{3}$
$=\frac{91}{6}\sqrt{3}$

39. Given, $4^{2x-1} - 16^{x-1} = 384$
$\Rightarrow 4^{2x-1} - 4^{2(x-1)} = 384$

$4^{2x-1}-\frac{4^{2x-2+1}}{4}=384$
$\Rightarrow 4^{2x-1}-\frac{4^{2x-1}}{4}=2^7\times 3$
$\Rightarrow 4^{2x-1}\left(1-\frac{1}{4}\right)=2^7\times 3$
$\Rightarrow 2^{2(2x-1)}\times\frac{3}{4}=2^7\times 3$
$\Rightarrow 2^{4x-2}=2^7\times 3\times\frac{2^2}{3}=2^9$
Equating the exponents, we get
$4x-2=9$
$\Rightarrow x=\frac{11}{4}.$

40. $\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}=a+\frac{7}{11}\sqrt{5}b$
$\frac{7+\sqrt{5}}{7-\sqrt{5}}\times\frac{7+\sqrt{5}}{7+\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}}\times\frac{7-\sqrt{5}}{7-\sqrt{5}}=a+\frac{7}{11}\sqrt{5}b$
$\frac{(7+\sqrt{5})^2}{(7)^2-(\sqrt{5})^2}-\frac{(7-\sqrt{5})^2}{(7)^2-(\sqrt{5})^2}=a+\frac{7}{11}\sqrt{5}b$
$\frac{49+5+14\sqrt{5}}{49-5}-\frac{49+5-14\sqrt{5}}{49-5}=a+\frac{7}{11}\sqrt{5}b$
$=\frac{54+14\sqrt{5}}{44}-\frac{54-14\sqrt{5}}{44}=a+\frac{7}{11}\sqrt{5}b$
$=\frac{54+14\sqrt{5}-54+14\sqrt{5}}{44}$
$=a+\frac{7}{11}\sqrt{5}b=\frac{28\sqrt{5}}{44}$
$\Rightarrow\frac{7\sqrt{5}}{11}=a+\frac{7}{11}\sqrt{5}b$
$\Rightarrow 0+\frac{7\sqrt{5}}{11}=a+\frac{7}{11}\sqrt{5}b$
Thus, a = 0 and b = 1.

41. $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}\times\frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ (Rationalizing by denominator)
$\frac{(7+3\sqrt{5})^2}{7^2-(3\sqrt{5})^2}=\frac{7^2+(3\sqrt{5})^2+2\times 7\times 3\sqrt{5}}{49-3^2\times 5}$
$=\frac{49+9\times 5+42\sqrt{5}}{49-45}=\frac{49+45+42\sqrt{5}}{4}$
$=\frac{94+42\sqrt{5}}{4}=\frac{94}{4}+\frac{42}{4}\sqrt{5}$
$=\frac{47}{2}+\frac{21}{2}\sqrt{5}$

42. $a = xy^{p-x}$, $b = xy^{q-1}$ and $c = xy^{r-1}$
$\therefore a^{q-r}\times b^{r-p}\times c^{p-q}$
$= (xy^{p-x})^{q-r}\times(xy^{q-1})^{r-p}\times(xy^{r-1})^{p-q}$
$= x^{q-r}\times y^{(p-1)(q-r)}\times x^{r-p}\times y^{(q-1)(r-p)}\times x^{p-q}\times y^{(r-1)(p-q)}$
$= x^{q-r}\times x^{r-p}\times x^{p-q}\times y^{pq-pr-q+r}\times y^{qr-pq-r+p}\times y^{pr-qr-p+q}$
$= x^{q-r+r-p+p-q}\times y^{p\ q-p\ r-q+r+q\ r-p\ q-r+p+p\ r-q\ r-p+q}$
$= x^0\times y^0$
$= 1\times 1$
$= 1$

43. $(-12)^3+(7)^3+(5)^3$
Let a = −12, b = 7 and c = 5
We know that, if $a+b+c=0, then, a^3+b^3+c^3=3abc$
Here, a+b+c=−12+7+5=0
$\therefore\ (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$
= - 1260

44. $5+2x$
We need to find the zero of the polynomial $5+2x$
$5+2x=0$
$\Rightarrow x=-\frac{5}{2}$

While applying the remainder theorem, we need to put the zero of the polynomial $5+2x$ in the polynomial x^3+3x^2+3x+1 ,to get

$p(x)=x^3+3x^2+3x+1$

$p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$

$=-\frac{125}{8}+3\left(\frac{25}{4}\right)-\frac{15}{2}+1$

$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$

$=\frac{-125+150-60+8}{8}$

$=-\frac{27}{8}$.

45. The given expression may be rewritten as,

$(a + b)^3 - [2(a - b)]^3$

$= (a + b)^3 - [2a - 2b]^3$

$= (a + b - (2a - 2b))((a + b)^2 + (a + b)(2a - 2b) + (2a - 2b)^2)$

$\therefore [a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + (a + b)(2a - 2b) + (2a - 2b)^2)$

$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + 2a^2 - 2ab + 2ab - 2b^2 + (2a - 2b)^2)$

$= (3b - a)(3a^2 + 2ab - b^2 + (2a - 2b)^2)$

$= (3b - a)(3a^2 + 2ab - b^2 + 4a^2 + 4b^2 - 8ab)$

$= (3b - a)(3a^2 + 4a^2 - b^2 + 4b^2 - 8ab + 2ab)$

$= (3b - a)(7a^2 +3b^2 - 6ab)$

$\therefore (a + b)^3 - 8(a - b)^3 = (3b - a)(7a^2 + 3b^2 - 6ab)$

46. Given

$\sqrt[3]{2}=2^{\frac{1}{3}};\sqrt{3}=3^{\frac{1}{2}};\sqrt[6]{5}=5^{\frac{1}{6}}$

LCM of 3,2 and 6 = 6

$\therefore \quad 2^{\frac{1}{3}}=2^{\left(\frac{1}{3}\times\frac{2}{2}\right)}=2^{\frac{2}{6}}=\left(2^2\right)^{\frac{1}{6}}=4^{\frac{1}{6}}$

$3^{\frac{1}{2}}=3^{\left(\frac{1}{2}\times\frac{3}{3}\right)}=3^{\frac{3}{6}}=\left(3^3\right)^{\frac{1}{6}}=(27)^{\frac{1}{6}}$

Clearly, $(27)^{\frac{1}{6}}>5^{\frac{1}{6}}>4^{\frac{1}{6}}$

So, $3^{\frac{1}{2}}>5^{\frac{1}{6}}>2^{\frac{1}{3}}$

or $\sqrt{3}>\sqrt[6]{5}>\sqrt[3]{2}$

Hence, the correct descending order is $\sqrt{3},\sqrt[6]{5}$ and $\sqrt[3]{2}$.

47. We know that

$\frac{9^n\times3^2\times\left(3^{-\frac{n}{2}}\right)^{-2}-(27)^n}{3^{3m}\times2^3}=\frac{1}{27}$

$\Rightarrow\frac{(3^2)^n\times3^2\times\left(3^{-\frac{n}{2}}\right)^{-2}-(3^3)^n}{3^{3m}\times2^3}=\frac{1}{3^3}$

$\Rightarrow\frac{(3)^{2n}\times3^2\times3^{\frac{n}{2}\times2}-(3)^{3n}}{3^{3m}\times2^3}=\frac{1}{3^3}$

$\Rightarrow\frac{(3)^{2n+2}\times3^n-(3)^{3n}}{3^{3m}\times2^3}=\frac{1}{3^3}$

$\Rightarrow\frac{(3)^{2n+2+n}-(3)^{3n}}{3^{3m}\times2^3}=\frac{1}{3^3}$

$\Rightarrow\frac{(3)^{3n+2}-(3)^{3n}}{3^{3m}\times2^3}=\frac{1}{3^3}$

$\Rightarrow 3^3\times\left[(3)^{3n+2}-(3)^{3n}\right]=3^{3m}\times2^3$

$\Rightarrow 3^{3+3n}\times\left[(3)^2-1\right]=3^{3m}\times2^3$

$\Rightarrow 3^{3+3n}\times[8]=3^{3m}\times2^3$

$\Rightarrow 3^{3+3n}\times2^3=3^{3m}\times2^3$

$\Rightarrow 3^{3+3n}=3^{3m}$

$\Rightarrow$ 3+3n = 3m

$\Rightarrow$ 3m - 3n = 3

$\Rightarrow$ m - n = 1

48. LHS

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

$= \frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{7\times3-7\sqrt{5}+3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2} - \frac{7\times3+7\sqrt{5}-3\sqrt{5}\times3-3\sqrt{5}\times\sqrt{5}}{3^2-\sqrt{5}^2}$

$= \frac{21-7\sqrt{5}+9\sqrt{5}-15}{9-5} - \frac{21+7\sqrt{5}-9\sqrt{5}-15}{9-5}$

$= \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$

$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$

$= \frac{0+4\sqrt{5}}{4}$

$= 0 + \sqrt{5}$

We know that,

$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = a + b\sqrt{5}$

$0 + \sqrt{5} = a + b\sqrt{5}$

a = 0 and b = 1

49. $\frac{3+\sqrt{5}}{3-\sqrt{5}}$

$= \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{(3+\sqrt{5})^2}{3^3-\sqrt{5}^2}$ [a^2 - b^2 = (a + b)(a - b)]

$= \frac{3^2+2\times3\sqrt{5}+\sqrt{5}^2}{9-5}$

$= \frac{9+6\sqrt{5}+5}{4}$

$= \frac{14+6\sqrt{5}}{4}$

$= \frac{7+3\sqrt{5}}{2}$

Substituting the value $\sqrt{5}$ we get,

$\frac{7+3\times2.236}{2}$

$= \frac{7+6.708}{2}$

$= \frac{13.708}{2}$

= 6.854

50. Let p(z) = $az^3 + 4z^2 + 3z - 4$

And q(z) = $z^3 - 4z + a$

As these two polynomials leave the same remainder, when divided by z - 3, then p(3) = q(3).

$\therefore$ p(3) = $a(3)^3 + 4(3)^2 + 3(3) - 4$

= 27a + 36 + 9 - 4

Or p(3) = 27a + 41

And q(3) = $(3)^3 - 4(3) + a$

= 27 - 12 + a = 15 + a

Now, p(3) = q(3)

$\Rightarrow$ 27a + 41 = 15 + a

$\Rightarrow$ 26a = -26; a = -1

Hence, the required value of a = -1.

51. p(x) $=2x^4 - 6x^3 + 2x^2 - x + 2$ and g(x) $= x + 2$

$$\begin{array}{r}
2x^3 - 10x^2 + 22x - 45 \\
x+2 \overline{\big) 2x^4 - 6x^3 + 2x^2 - x + 2} \\
\underline{\overset{-}{2x^4} + \overset{-}{4x^3}} \\
-10x^3 + 2x^2 \\
\underline{\overset{+}{-10x^3} \overset{+}{-20x^2}} \\
22x^2 - x \\
\underline{\overset{-}{22x^2} + \overset{-}{44x}} \\
-45x + 2 \\
\underline{\overset{+}{-45x} \overset{+}{-90}} \\
\underline{92}
\end{array}$$

Quotient = $2x^3 - 10x^2 + 22x - 45$

Remainder = 92

Verification: Divisor $\times$ Quotient + Remainder

$= (x + 2) \times (2x^3 - 10x^2 + 22x - 45) + 92$

$= x (2x^3 - 10x^2 + 22x - 45) + 2(2x^3 - 10x^2 + 22x - 45) + 92$

$= 2x^4 - 10x^3 + 22x^2 - 45x + 4x^3 - 20x^2 + 44x - 90 + 92$

$= 2x^4 - 6x^3 + 2x^2 - x + 2$

= Dividend

Therefore,division algorithm is verified.

52. Here, $f(x) = ax^3 + x^2 - 2x + b$

(x + 1) and (x - 1) are the factors

From factor theorem, if(x- 1), (x+1) are the factors of f(x) then f(1) = 0 and f(-1) = 0

Let, x - 1= 0

$\Longrightarrow$ x = 1

Substitute x=1 in f(x),then,we have,

$f(1) = a(1)^3 + (1)^2 - 2(1) + b$

= a + 1 - 2 + b

= a + b - 1

Since f(1)=0, therefore

a+b-1=0 ....(1)

Let, x + 1 = 0

$\Longrightarrow$ x = -1

Substitute x=-1 in f(x),then,we have,

$f(-1) = a(-1)^3 + (-1)^2 - 2(-1) + b$

= -a + 1 + 2 + b

= -a + b + 3

Since f(-1)=0, therefore,we have,

-a+b+3=0 ....(2)

Solve equations 1 and 2

a + b = 1

-a + b = -3

2b = -2

$\Longrightarrow$ b = -1

substitute b value in eq 1

$\Longrightarrow$ a - 1 = 1

$\Longrightarrow$ a = 1 + 1

$\Longrightarrow$ a = 2

The values are a= 2 and b = -1

53. We have, p(x) = $8x^3 - ax^2 - x + 2$

Since x=$-\frac{1}{2}$ is a zero of p(x), therefore $p\left(-\frac{1}{2}\right) = 0$

Substitute the value of x in p(x)

p($-\frac{1}{2}$) = 8($-\frac{1}{2}$)^3 - a($-\frac{1}{2}$)^2 - ($-\frac{1}{2}$) + 2

= - 8($\frac{1}{8}$) - a($\frac{1}{4}$) + $\frac{1}{2}$ + 2

= -1 - ($\frac{a}{4}$ + $\frac{1}{2}$ + 2)

= 1 - ($\frac{a}{4}$ + $\frac{1}{2}$)

= $\frac{3}{2} - \frac{a}{4}$

To, find the value of a, equate p($-\frac{1}{2}$) to zero

p($-\frac{1}{2}$) = 0

$\frac{3}{2} - \frac{a}{4}$ = 0

On taking L.C.M

$\frac{6-a}{4} = 0$

$\Longrightarrow$ 6 - a = 0

$\Longrightarrow$ a = 6