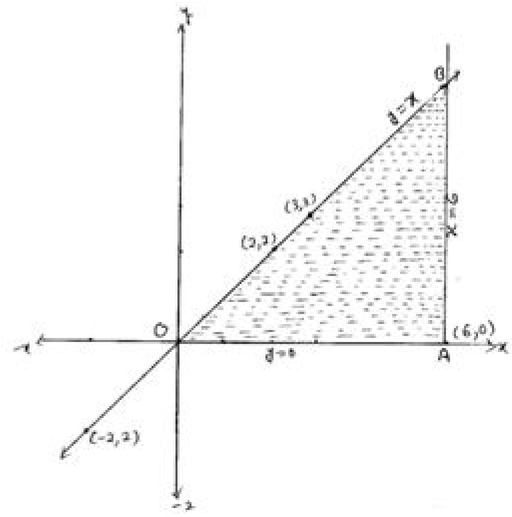
Solution

Class 10 - Mathematics

MATHEMATICS

1. **(b)** x^2 - 3x -10
   Explanation: Given, sum and the product of the zeros of a quadratic polynomial are 3 and -10 respectively i,e. $(\alpha + \beta) = 3$ and $\alpha\beta = -10$
   Therefore, required polynomial is
   $x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$
2. **(c)** c and a have the same sign
   Explanation: If the zeroes of a quadratic polynomial $ax^2 + bx + c, c \neq 0$are equal, then $b^2 - 4ac = 0$.
   $\implies b^2 = 4ac$. Here b^2 is always positive
   And this is possible only if a and c are both positive or both negative. Hence both should have the same sign
3. **(d)** -5
   Explanation: $x^2 + 5x + 8$
   $\alpha + \beta = \frac{-\text{Coefficient of x}}{\text{Coefficient of } x^2}$
   $= \frac{-5}{1}$
   = -5
4. **(a)** x = 3, y = 2
   Explanation: We have:
   2x + 3y = 12 ...(i)
   3x – 2y = 5 ...(ii)
   Now, by multiplying (i) by 2 and (ii) by 3 and then adding them we get:
   4x + 9x = 24 + 15
   13x = 39
   x = $\frac{39}{13}$ = 3
   Now putting the value of x in (i), we get
   2 × 3 + 3y = 12
   $\therefore$ y = $\frac{12-6}{3}$ = 2
5. **(a)** 18 sq. units
   Explanation: The triangle formed by the lines y = x, x = 6 and y = 0 is shaded.
   The area of the shaded region, i.e., $x = y$
   We got a right-angled triangle with base 6 units and height 6 units
   Triangle OAB = $\frac{1}{2} \times$ OA $\times$ AB
   Hence area of triangle =(1/2) $\times$ 6 $\times$ 6 = 18 sq units
   = $\frac{1}{2} \times 6 \times 6$ = 18 sq.units

x	2	-2	3
y	2	-2	3



6. **(d)** 3200
Explanation: In an A.P.
a = 2 and d = 4, n = 40
$\therefore S_n = \frac{n}{2}[2a + (n-1)d] = \frac{40}{2}[2 \times 2 + (40-1) \times 4]$
$= 20[4 + 39 \times 4] = 20 \times (4 + 156) = 20 \times 160 = 3200$

7. **(b)** 13
Explanation: For 1st A.P.; a = 63 and d = 2 , hence T_n = 63 + (n - 1)2
$\Rightarrow$ T_n = 2n + 61 ... (i)
for 2nd AP a = 3, d = 7, hence t_n = 3 + (n - 1)7
t_n = 7n - 4 ... (ii)
by condition, from (i) & (ii),
7n - 4 = 2n + 61
$\Rightarrow$ 5n = 65
$\therefore$ n = 13

8. **(c)** cos $\frac{A}{2}$
Explanation: We know that
$\angle$A + $\angle$B + $\angle$C = 180°
$\Rightarrow \frac{\mathrm{A+B+C}}{2} = 90^\circ$
$\Rightarrow \frac{\mathrm{A}}{2} + \frac{\mathrm{B+C}}{2} = 90^\circ$
$\Rightarrow \frac{\mathrm{B+C}}{2} = 90^\circ - \frac{\mathrm{A}}{2}$
$\Rightarrow \sin\left(\frac{\mathrm{B+C}}{2}\right) = \sin\left(90^\circ - \frac{\mathrm{A}}{2}\right)$
$\Rightarrow \sin\left(\frac{\mathrm{B+C}}{2}\right) = \cos\frac{\mathrm{A}}{2}$

9. **(b)** 1
Explanation: Given that, sin A + $\sin^2$ A = 1
$\Rightarrow$ sin A = 1 - $\sin^2$ A
$\Rightarrow$ sin A = $\cos^2$ A
$\Rightarrow$ $\sin^2$ A = $\cos^4$ A
$\Rightarrow$ 1 - $\cos^2$ A = $\cos^4$ A
$\Rightarrow$ $\cos^2$ A + $\cos^4$ A = 1

10. **(c)** $sec\,\theta + tan\,\theta$
Explanation: Given: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$
= $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$

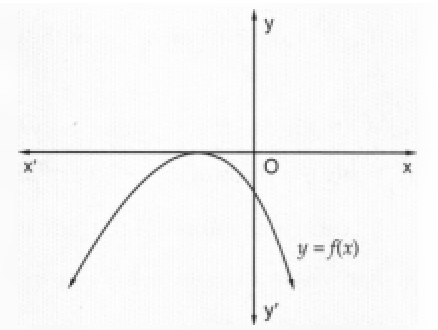
= $\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

= $\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

= $\frac{1+\sin\theta}{\cos\theta}$

= $\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$

= $\sec\theta+\tan\theta$

11. We have function p(x) = ax^2 + bx + c and a + c = b
    using remainder theorem by putting x = -1 we get
    p (-1) = $a(-1)^2$+ b(- 1) + c
    = a - b + c = a + c - b
    = b - b = 0
    $\therefore$ One zero is -1.
12. We have,
    f(x) = $6x^2$ - 3
    = $3(2x^2 - 1)$
    $\therefore$ f(x) = 0
    $\Rightarrow 3(2x^2 - 1) = 0$
    $\Rightarrow 2x^2 - 1 = 0$
    $\Rightarrow x^2 = \frac{1}{2}$
    $\Rightarrow x = \pm\frac{1}{2}$
    So, the zeros of f(x) are $\frac{1}{2}$ and $-\frac{1}{2}$
13. Let p(x) = $6x^2$ - 18x + 12
    For zeroes of p(x), we put p(x) = 0
    $6x^2$ - 18x + 12 = 0
    $6x^2$ - 6x - 12x + 12 = 0
    6x(x - 1) - 12(x - 1) = 0
    (6x - 12)(x - 1) = 0
    Either 6x - 12 = 0 or x - 1 = 0
    Either x = 2 or x = 1
    Thus, the zeroes of p(x) are 1, 2.
    Now, sum of zeros = 1 + 2 = 3 $= -\frac{(-18)}{6} = -\frac{coeff.\ of\ x}{coeff.\ of\ x^2}$
    and product of zeros = 1$\times$ 2 = 2 $= \frac{12}{6} = \frac{constant\ term}{coeff.\ of\ x^2}$
14. The graph of the polynomial $f(x) = ax^2 + bx + c$ touches x−axis at one point. And we know that if the graph intersects or touches the X-axis at exactly one point then a quadratic polynomial has two equal zeroes.
    

    Hence the number of real zeros of f(x) is 2 and b^2 - 4 ac = 0.
15. Since α, β are the zeros of the polynomial f(x) = x^2 - 5x + k.
    Compare f(x) = x^2 - 5x + k with ax^2 + bx + c.
    So, a = 1 , b = -5 and c = k
    $\alpha + \beta = -\frac{(-5)}{1}$ = 5

$\alpha\beta = \frac{k}{1} = k$

Given, $\alpha - \beta$ = 1

Now, $(\alpha+\beta)^2 = (\alpha-\beta)^2 + 4\alpha\beta$

$\Rightarrow$ $(5)^2 = (1)^2 + 4k$

$\Rightarrow$ 25 = 1 + 4k

$\Rightarrow$ 4k = 24

$\Rightarrow$ k = 6

Hence the value of k is 6.

16. Let $p(x) = x^4 - 3x^3 + 6x - 4$

Since $\sqrt{2}, -\sqrt{2}$ are zeros of p(x), therefore, $(x-\sqrt{2})(x+\sqrt{2})$ is a factor of p(x)

i.e (x^2-2) is a factor of p(x).

Let us divide p(x) by $x^2 - 2$

$x^2 - 2 \,)\, x^4 - 3x^3 + 6x - 4 \,(\, x^2 - 3x + 2$

$x^4 \quad - 2x^2$

$- \quad +$

$- 3x^2 + 2x^2 + 6x - 4$

$- 3x^3 \quad + 6x$

$+ \quad -$

$2x^2 - 4$

$2x^2 - 4$

$- \quad +$

0

By division algorithm, we have,

Dividend = Divisor $\times$ quotient + remainder

$x^4 - 3x^3 + 6x - 4 = (x^2 - 2)(x^2 - 3x + 2) + 0$

$x^4 - 3x^3 + 6x - 4 = (x^2 - 2)(x^2 - 3x + 2)$

For other two zeros of p(x), we put,

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

x(x-2) - 1(x-2) = 0

(x - 1)(x - 2) = 0

x = 1, 2

Thus the other two zeros are 1, 2.

17. We know, quadratic polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

Given, Sum of zeroes = $-\frac{21}{8}$ and Product of zeroes = $\frac{5}{16}$

$\therefore$ Quadratic Polynomial = $x^2 + \frac{21}{8}x + \frac{5}{16}$

= $\frac{1}{16}(16x^2 + 42x + 15)$

$\Rightarrow$ Quadratic polynomial is $16x^2 + 42x + 15$

Now, we rewrite the polynomial as $16x^2 + 2x + 40x + 5$

= $2x \cdot (8x+1) + 5 \cdot (8x+1)$

= $(2x+5) \cdot (8x+1)$

Now, for Zeros, $(8x+1) \cdot (2x+5) = 0$

$\Rightarrow x = \frac{-1}{8}, \frac{-5}{2}$

18. The given polynomial is p(x) = $6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6$

and q(x) = $2x^2 - 3$

$$
\begin{array}{r l}
 & 3x^3 + 2x^2 - 9x - \frac{1}{2} \\ \hline
2x^2 - 3 \,\big) & 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6 \\
 & 6x^5 \quad - 9x^3 \\
 & - \quad\quad + \\ \hline
 & 4x^4 - 18x^3 - 7x^2 - 27x - 6 \\
 & 4x^4 \quad - 6x^2 \\
 & - \quad\quad + \\ \hline
 & -18x^3 - x^2 - 27x - 6 \\
 & -18x^3 \quad + 27x \\
 & + \quad\quad - \\ \hline
 & -x^2 - 54x - 6 \\
 & -x^2 \quad + \frac{3}{2} \\
 & + \quad\quad - \\ \hline
 & -54x - \frac{15}{2} \\ \hline
\end{array}
$$

So, g(x) = $3x^3 + 2x^2 - 9x - \frac{1}{2}$

and r(x) = $-54x - \frac{15}{2}$

19. Here, $p(x) = 6x^4 - 44x^2 + 6x - 3$and $q(x) = x^2 - 3x + 1$ both are in standard form.

Now, $p(x)$ on dividing by $q(x)$, we get the following division process.

$$
\begin{array}{r l}
 & 6x^2 + 18x + 4 \\ \hline
x^2 - 3x + 1 \,\big) & 6x^4 + 0x^3 - 44x^2 + 6x - 3 \\
 & 6x^4 - 18x^3 + 6x^2 \\
 & - \quad + \quad - \\ \hline
 & 18x^3 - 50x^2 + 6x - 3 \\
 & 18x^3 - 54x^2 + 18x \\
 & - \quad + \quad - \\ \hline
 & 4x^2 - 12x - 3 \\
 & 4x^2 - 12x + 4 \\
 & - \quad + \quad - \\ \hline
 & -7 \\ \hline
\end{array}
$$

So, $q(x) = 6x^2 + 18x + 4$ and $r(x) = -7$

Verification

Divisior $\times$ Quotient + remainder

= $g(x) \times q(x) + r(x) = (x^2 - 3x + 1)(6x^2 + 18x + 4) - 7$

$= 6x^4 - 44x^2 + 6x + 4 - 7$

$= 6x^4 - 44x^2 + 6x - 3 = p(x) = Dividend$

Hence, the division algorithm is verified.

20. Given pair of linear equations is

- x + py = 1 ...(i)

and px - y - 1 = 0 ...(ii)

On comparing with standard form, we get

a_1 = - 2, b_1 = p, c_1 =- 1;

And a_2 = p, b_2 = - 1, c_2 =- 1;

a_1 /a_2 = $-\frac{1}{p}$

b_1 /b_2 = - p

c_1 /c_2 = 1

Since, the lines equations has no solution i.e., both lines are parallel to each other.

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$-\frac{1}{p}$ = - p $\neq$ 1

Taking last two parts, we get

p $\neq$ - 1

Taking first two parts, we get

$p^2 = 1$

$p = \pm 1$

Hence, the given pair of linear equations has no solution for p = 1.

21. Let the common ratio term of income be x and expenditure be y.

    So, the income of first person is Rs.9x and the income of second person is Rs.7x.

    And the expenditures of first and second person is 4y and 3y respectively.

    Then, Saving of first person =9x - 4y

    and saving of second person = 7x - 3y

    As per given condition

    9x - 4y = 200

    $\Rightarrow$ 9x - 4y - 200=0 ... (i)

    and, 7x - 3y = 200

    $\Rightarrow$ 7x - 3y - 200 =0 ..... (ii)

    Solving equation (i) and (ii) by cross-multiplication, we have

    $\frac{x}{800-600} = \frac{-y}{-1800+1400} = \frac{1}{-27+28}$

    $\frac{x}{200} = \frac{-y}{-400} = \frac{1}{1}$

    $\Rightarrow$ x =200 and y =400

    So, the solution of equations is x = 200 and y = 400.

    Thus, monthly income of first person = Rs.9x = Rs.(9 $\times$ 200)= Rs.1800

    and, monthly income of second person = Rs.7x = Rs.(7$\times$ 200)= Rs. 1400

22. The given equations are

    x + 2y + 1 = 0 ....(i)

    2x - 3y - 12 = 0 ....(ii)

    By cross multiplication, we have

    $\therefore \frac{x}{[2(-12)-1\times(-3)]} = \frac{y}{[1\times2-1\times(-12)]} = \frac{1}{[1\times(-3)-2\times2]}$

    $\Rightarrow \frac{x}{(-24+3)} = \frac{y}{(2+12)} = \frac{1}{(-3-4)}$

    $\Rightarrow \frac{x}{-21} = \frac{y}{14} = \frac{1}{-7}$

    $\Rightarrow \frac{x}{-21} = \frac{1}{-7}, \frac{y}{14} = \frac{1}{-7}$

    $\Rightarrow x = \frac{-21}{-7} = 3, \quad y = \frac{14}{-7} = -2$

    Hence, x = 3 and y = -2 is the solution of give equations.

23. Given equations are

    x + (k + 1)y = 4

    (k +1) x + 9y = 5k + 2

    We know that the system of equations

    $a_1x + b_1y = c_1$

    $a_2x + b_2y = c_2$

    has infinitely many solutions if

    $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

    Here,

    $\frac{1}{k+1} = \frac{(k+1)}{9} = \frac{4}{5k+2}$

    $\Rightarrow \frac{1}{k+1} = \frac{(k+1)}{9}$

    $\Rightarrow 9 = (k+1)^2$

    $\Rightarrow 3^2 = (k+1)^2$

    $\Rightarrow k + 1 = 3$

    $\Rightarrow k = 2$

    Hence for k = 2 the system of equation have infinitely many solutions.

24. The given pair of the equation is

    $\frac{4}{x} + 3y = 14$ ....(1)

    $\frac{3}{x} - 4y = 23$ ....(2)

Put $\frac{1}{x} = x$ ....(3)
Then equation (1) and (2) can be rewritten as
4x + 3y = 14 ...(4)
3x - 4y = 23 ...(5)
From equation (5),
4y = 3x - 23
$\Rightarrow \; y = \frac{3x-23}{4}$ ....(6)
Substituting this value of y in equation (4) we get
$4x + 3\left(\frac{3x-23}{4}\right) = 14$
⇒ 25x = 56 + 69 = 125
$\Rightarrow \; x = \frac{125}{25} = 5$ ....(7)
Substituting this value of x in equation (6), we get
$y = \frac{3(5)-23}{4} = \frac{15-23}{4}$
$= \frac{-8}{4} = -2$ ...(8)
From equation (3) and equation (7), we get $\frac{1}{x} = 5$
$\Rightarrow \; \frac{1}{x} = 5$
Hence, the solution of the given pair of the equation is,$\frac{1}{x} = 5$ y = -2.
Verification. Substituting $\frac{1}{x} = 5$, y = -2,
We find that both the equations (1) and (2) are satisfied as shown below:
$\frac{4}{x} + 3y = \frac{4}{\left(\frac{1}{5}\right)} + 3(-2) = 20 - 6 = 14$
$\frac{3}{x} - 3y = \frac{3}{\left(\frac{1}{5}\right)} - 4(-2) = 15 + 8 = 23$
Hence, the solution is correct.

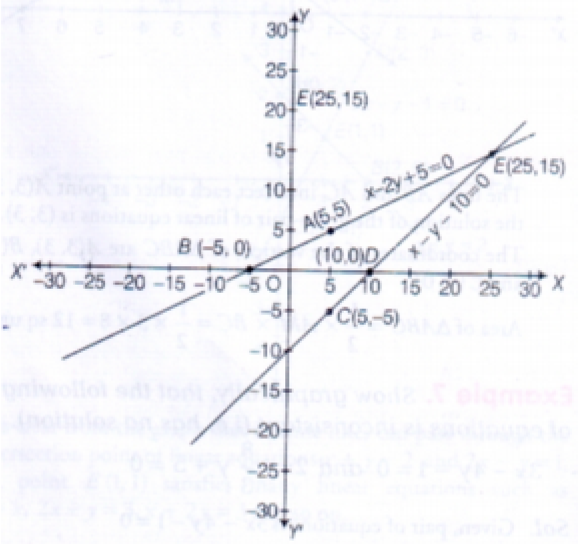
25. Let the present age of Sagar be x years and the age of Tiru be y year.
5 years ago, Sagar's age = $(x-5)$years and Tiru's age =$(y-5)$years
According to given condition,
$(x-5) = 2(y-5)$
$\Rightarrow x - 5 = 2y - 10 \Rightarrow x - 2y + 5 = 0$
After 10 yr, Sagar's age = $(x+10)$yrs and Tiru's age =$(y+10)$yrs
According to the given question,
$x + 10 = (y+10) + 10$
$\Rightarrow x + 10 = y + 20 \Rightarrow x - y - 10 = 0$
Thus, we get following pair of linear equations
$\Rightarrow x - 2y + 5 = 0$ ....(i)
$\Rightarrow x - y - 10 = 0$ ...(ii)
Now, Let us draw the graphs of Eqs.(i) and (ii) , by finding atleast two solutions of each of the above equations. The solutions of equations are given in the following tables.
Table for $x - 2y + 5 = 0$

x	5	-5
$y = \frac{x+5}{2}$	5	0
Points	A(5,5)	B(-5,0)

Table for $x - y - 10 = 0$

x	5	10
y = x - 10	-5	0
Points	C(-5,5)	D(10,0)

Plot the points A(5,5) and B(-5,0) on a graph paper and join them to get the line AB. Similarly, plot the points C(-5,5) and D(10,0) on the same graph paper and join them to get line CD.



It is clear from the graph that, lines AB and CD intersect each other at point E(25,15).

So, $x = 25$ and $y = 15$ is the required solution.

Hence, Sagar's present age = 25 yr and Tiru's present age = 15 yr

26. Let $\frac{1}{3x+2y} = u$

and $\frac{1}{3x-2y} = v$

$\Rightarrow 2u + 3v = \frac{17}{5}$

and 5u + v = 2

on solving, we get $u = \frac{1}{5}$

and v = 1

$\therefore$ 3x + 2y = 5

3x - 2y = 1

On solving, we get

x = 1 and y = 1

27. Let a be the first term and d be the common difference of the given AP. Then,

$S_n = \frac{n}{2} \cdot [2a+(n-l)d]$,

$\therefore 3(S_8 - S_4) = 3[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)]$

= $3[4(2a+7d) - 2(2a+3d)] = 6(2a+11d)$

$= \frac{12}{2} \cdot (2a+11d) = S_{12}$.

Hence, $S_{12} = 3(S_8 - S_4)$.

28. We are given that

a = 1

$a_n = 11$

$s_n = 36$

36 = $\frac{n}{2}$ {(a + a_n)} { using formula for sum to n terms}

36 = $\frac{n}{2}$ [12]

72 = 12 n

$n = \frac{72}{12}$

n = 6

29. let the first term, common difference of an AP are a and d respectively.

a = 7

d = $a_2 - a_1$ = 10 - 7 = 3

Let the nth term of this AP is 55

Then

$a_n = 55$

a + (n - 1)d = 55

7 + (n - 1)3 = 55

3(n - 1) = 48
n - 1 = 16
n = 17
so the 55 is a term of given AP
and 55 is the 17^{th} term of given AP

30. For the said AP it is given that
$\mathrm{a}=5, \mathrm{d}=3$ and $\mathrm{a_n}=80$
$\Rightarrow$ a + (n - 1)d = 80
$\Rightarrow 5+(n-1)\times 3=80$
$\Rightarrow$ 5 + 3n - 3 = 80
$\Rightarrow$ 3n = 80 + 3 - 5
$\Rightarrow$ 3n = 78
$\Rightarrow \quad n=\frac{78}{3}=26$
Therefore, Number of terms = 26
31. Let the first term be a and the common difference be d.
a_n = a + (n - 1)d
Here given, a_3 =9
or, a + 2d = 9 ....(i)
a_8 - a_5 = 6
or, (a + 7d) - (a + 4d) = 6
a + 7d - a - 4d = 6
or, 3d = 6
or, d = 2 ....(ii)
Substituting this value of d from (ii) in (i), we get
or, a + 2(2) = 9
or, a + 4 = 9
or a = 9 - 4
or, a = 5
a = 5 and d = 2
So, A.P. is 5,7,9,11,....
32. According to question we observe that 56 is the first integer between 50 and 500 which is divisible by 7. Also, when we divide 500 by 7 the remainder is 3. Therefore, 500 - 3 = 497 is the largest integer divisible by 7 and lying between 50 and 500. Thus, we have to find the number of terms in an A.P. with first term = 56, last term = 497 and common difference = 7 (as the numbers are divisible by 7).
Let there be n terms in the A.P. Then,
a_n = 497
$\Rightarrow$ a + (n - 1) d = 497
$\Rightarrow \quad 56+(n-1)\times 7=497$ ...[Because,a = 56 and d = 7]
$\Rightarrow$ 7n + 49 = 497
$\Rightarrow$ 7n = 448
$\Rightarrow$ n = 64
Threfore, there are 64 integers between 50 and 500 which are divisible by 7.
33. The multiples of 4 that lie between 10 and 250 are:
12, 16, 20, 24, ..., 248
a_2 - a_1 = 16 - 12 = 4
a_3 - a_2 = 20 - 16 = 4
a_4 - a_3 = 24 - 20 = 4
As a_{k+1} - a_k is the same for k = 1, 2, 3, etc.
The above list of numbers forms an AP with the first term a = 12
and the common difference d = 4
Last term (l) = 248
Let there be n term s in this AP. Then, nth term = l

$\Rightarrow$ a + (n - 1)d = 248
$\Rightarrow$ 12 + (n - 1)4 = 248
$\Rightarrow$ (n - 1)d = 248 - 12
$\Rightarrow$ (n - 1) = 236
$\Rightarrow n-1=\frac{236}{4}$
$\Rightarrow$ n - 1 = 59
$\Rightarrow$ n = 59 + 1
$\Rightarrow$ n = 60
Hence, 60 multiples of 4 lie between 10 and 250.

34. Using Sec(90-θ)= Cosecθ & Cosec(90 - θ) =Secθ in the given expression , we get :-
sec 50° sin 40° + cos 40° cosec 50°
= sec (90° - 40°) sin 40° + cos 40° cosec (90° - 40°)
= cosec 40° sin 40° + cos40° sec 40°
$=\frac{\sin 40^{\circ}}{\sin 40^{\circ}}+\frac{\cos 40^{\circ}}{\cos 40^{\circ}}$
= 1 + 1 = 2

35. We know that, tan45° = 1 = Cot45° = Sin90° = Cos0° , Cosec30° = 2 = Sec 60°, putting these values in the given expression , we get :-
$\frac{\tan 45^{\circ}}{cosec 30^{\circ}}+\frac{\sec 60^{\circ}}{\cot 45^{\circ}}-\frac{5 \sin 90^{\circ}}{2 \cos 0^{\circ}}$
$=\frac{1}{2}+\frac{2}{1}-\frac{5\times1}{2\times1}$
$=\frac{1}{2}+\frac{2}{1}-\frac{5}{2}$
$=\frac{1+4-5}{2}$
$=\frac{0}{2}=0$

36. LHS = (1 - cos$^2\theta$)sec$^2\theta$
= sin$^2\theta\times$ sec$^2\theta$ [(1 - cos$^2\theta$) = sin$^2\theta$]
= $\sin^2\theta\times\frac{1}{\cos^2\theta}=\frac{\sin^2\theta}{\cos^2\theta}$
= tan$^2\theta$ = RHS
Hence Proved

37. LHS = (sin α + cos α) (tan α + cot α)
= (sin α + cos α) $\left(\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}\right)$
= (sin α + cos α) $\left(\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha\cos\alpha}\right)$
= (sin α + cos α) $\frac{1}{\sin\alpha\cos\alpha}$ [$\because$ sin^2 α + cos^2 α = 1]
$=\frac{\sin\alpha}{\sin\alpha\cos\alpha}+\frac{\cos\alpha}{\sin\alpha\cos\alpha}$
$=\frac{1}{\cos\alpha}+\frac{1}{\sin\alpha}$
= sec α+cosec α
= RHS
Hence, proved.

38. We have,
$2\left(\frac{\cos 58^{\circ}}{\sin 32^{\circ}}\right)-\sqrt{3}\left(\frac{\cos 38^{\circ} cosec 52^{\circ}}{\tan 15^{\circ}\tan 60^{\circ}\tan 75^{\circ}}\right)$
$=2\left\{\frac{\cos(90^{\circ}-32^{\circ})}{\sin 32^{\circ}}\right\}-\sqrt{3}\left\{\frac{\cos 38^{\circ} cosec(90^{\circ}-38^{\circ})}{\tan 15^{\circ}\tan 60^{\circ}\tan(90^{\circ}-15^{\circ})}\right\}$
$=2\left(\frac{\sin 32^{\circ}}{\sin 32^{\circ}}\right)-\sqrt{3}\left\{\frac{\cos 38^{\circ}\sec 38^{\circ}}{\tan 15^{\circ}\times\sqrt{3}\times\cot 15^{\circ}}\right\}$
$[\because\ \cos(90-\theta)=\sin\theta\ ,\ \cos ec\,(90-\theta)=sec\theta,\ \tan(90-\theta)=cot\theta\ ]$
$=2-\sqrt{3}\left\{\frac{\cos 38^{\circ}\times\frac{1}{\cos 38^{\circ}}}{\tan 15^{\circ}\times\sqrt{3}\times\frac{1}{\tan 15^{\circ}}}\right\}=2-\frac{\sqrt{3}}{\sqrt{3}}=2-1=1\left[sec\theta=\frac{1}{\cos\theta},\ cot\theta=\frac{1}{\tan\theta}\right]$
therefore, $2\left(\frac{\cos 58^{\circ}}{\sin 32^{\circ}}\right)-\sqrt{3}\left(\frac{\cos 38^{\circ} cosec 52^{\circ}}{\tan 15^{\circ}\tan 60^{\circ}\tan 75^{\circ}}\right)=1$

39. cos 60° = 1/2 $\Rightarrow$ cos^2 60° = 1/4
sin 45° = 1/$\sqrt{2}\Rightarrow$ sin^2 45° = 1/2

sin 30° = 1/2 ⇒ $\sin^2 30°$ = 1/4

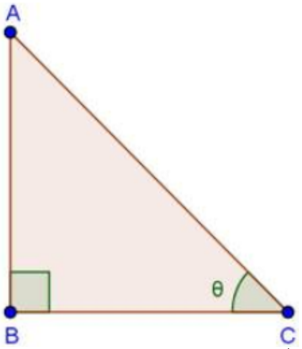
cos 90° = 0 ⇒ $\cos^2 90°$ = 0

∴ 2 $\cos^2 60°$ + 3 $\sin^2 45°$ - 3 $\sin^2 30°$ + 2 $\cos^2 90°$

= 2(1/4) + 3(1/2) - 3(1/4) + 2(0)

= (1/2) + (3/2) - (3/4) = 2 - (3/4)

= 5/4

40. 

Given $\tan\theta = \frac{1}{\sqrt{7}} = \frac{AB}{BC}$

Let AB = 1K

and, $BC = \sqrt{7}K$

In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$$AC^2 = (1K)^2 + \left(\sqrt{7}K\right)^2$$

$AC^2 = K^2 + 7K^2$

$AC^2 = 8K^2$

$$AC = \sqrt{8K^2} = 2\sqrt{2}K$$

$$\therefore \cos ec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}K}{1K} = 2\sqrt{2}$$

$$\sec\theta = \frac{AC}{BC} = \frac{2\sqrt{2}K}{\sqrt{7}K} = \frac{2\sqrt{2}}{\sqrt{7}}$$

$$\therefore LHS = \frac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \sec^2\theta}$$

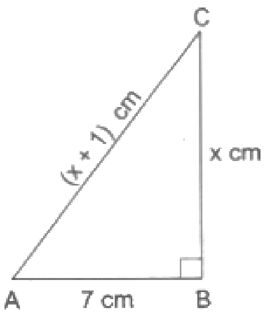
$$= \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{7}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$

$$= \frac{\frac{48}{7}}{\frac{64}{7}}$$

$$= \frac{48}{7} \times \frac{7}{64} = \frac{3}{4} = RHS$$

41. 

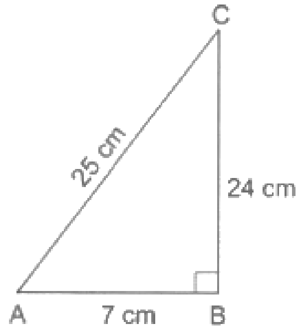
Let BC = x cm . Then, AC = (x + 1) cm.

By Pythagoras' theorem, we have

$AB^2 + BC^2 = AC^2$

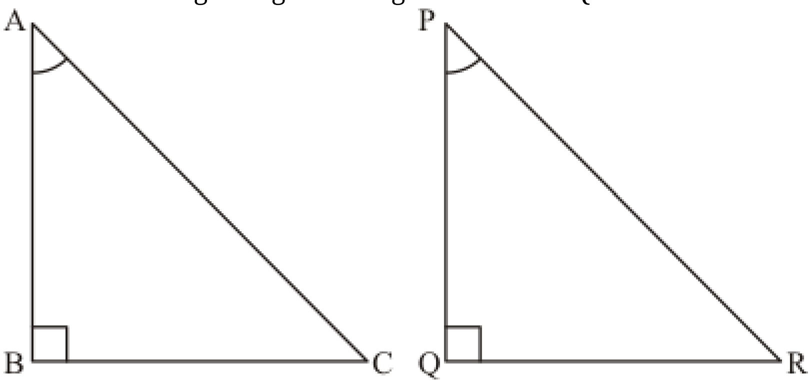
⇒ $7^2 + x^2 = (x + 1)^2$

$\Rightarrow 49 + x^2 = x^2 + 2x + 1$
$\Rightarrow 2x = 48$
$x = 24$.
$\therefore$ BC = 24 cm, AC = 25 cm and AB = 7 cm.



For T-ratios of $\angle$A, we have
$\sin A = \frac{BC}{AC} = \frac{24}{25}$ and $\cos A = \frac{AB}{AC} = \frac{7}{25}$
For T-ratios of $\angle$C, we have
$\sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$

42. Given: tan A = tan P
To show: $\angle A = \angle P$
Consider two right angled triangles ABC and PQR such that tan A = tan P



Therefore we have,
$\tan A = \frac{BC}{AB}$ and $\tan P = \frac{QR}{PQ}$
Since it is given that tan A = tan P
Therefore,
$\frac{BC}{AB} = \frac{QR}{PQ}$
Now by interchanging position of *AB* and *QR* by cross multiplication
We get,
$\frac{BC}{QR} = \frac{AB}{PQ}$
Let $\frac{BC}{QR} = \frac{AB}{PQ} = k$ (say) ...(1)
Now by cross multiplication
BC = kQR and AB = kPQ ...(2)
Now by using Pythagoras theorem in triangles ABC and PQR
We have,
$AC^2 = AB^2 + BC^2$ and $PR^2 = PQ^2 + QR^2$
Therefore
$AC = \sqrt{AB^2 + BC^2}$ and $PR = \sqrt{PQ^2 + QR^2}$
Now $\frac{AC}{PR} = \frac{\sqrt{AB^2+BC^2}}{\sqrt{PQ^2+QR^2}}$
Now using equation (2)
We get,

$\frac{AC}{PR} = \frac{\sqrt{(kPQ)^2+(kQR)^2}}{\sqrt{PQ^2+QR^2}}$

$\frac{AC}{PR} = \frac{\sqrt{k^2PQ^2+k^2QR^2}}{\sqrt{PQ^2+QR^2}}$

We get, $\frac{AC}{PR} = \frac{\sqrt{k^2(PQ^2+QR^2)}}{\sqrt{PQ^2+QR^2}}$

Therefore,

$\frac{AC}{PR} = \frac{k\sqrt{(PQ^2+QR^2)}}{\sqrt{PQ^2+QR^2}}$

Now $\sqrt{PQ^2+QR^2}$ gets cancelled

Therefore,

$\frac{AC}{PR} = k$ ... (3)

From (1) and (3)

$\frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR} = k$

Therefore, $\triangle ABC \sim \triangle PQR$

Hence, $\angle A = \angle P$

43. i. (b) -10
    ii. (c) 1
    iii. (c) -4, 3
    iv. (d) x^2 - 16
    v. (c) p(x) = x^2
44. i. (b) 4000,5000,6000,...
    ii. (c) 4000 and 1000
    iii. (c) 15 days
    iv. (d) 13000
    v. (d) 28